

4.3 Mixing

In cooking we mix ingredients, and many traditional manufacturing processes start by mixing raw materials; moreover, many mixtures (e.g. soil) are vitally important in the natural world. But here I mean “mixing to achieve randomness”. Perhaps the most familiar example is card shuffling, and I’ll describe a little of the mathematics of card shuffling below, but let me start with an analogy. The fact

ice floats on water

we observe in childhood. Not until taking a chemistry course do we learn this is really a remarkable special feature of water; most liquids become denser upon freezing, so the frozen substance would sink beneath the liquid substance. Analogously, most of us learn in later childhood

playing cards are easy to mix by shuffling.

But we rarely pause to consider that this is really a remarkable special feature of playing cards. Though there has been surprisingly little serious research, I am willing to assert that, for most physical objects used for randomness, to make them genuinely random you need to do much more mixing than you would intuitively think.

A famous example, discussed in many textbooks in mathematical statistics, was the 1970 draft lottery. To quote [39], which gives both the raw data and results of several different statistical analysis,

In an attempt to expose male youth fairly to the risk of being drafted, a lottery was held to allocate birthdates at random: 366 capsules, each containing a unique day of the year, were successively drawn from a container. The first date drawn (September 14) was assigned rank 1, the second date drawn (April 24) was assigned rank 2, and so on. Those eligible for the draft who were born on September 14 were called first for physicals, then those born on April 24 were tapped, and so on.

This lottery was a source of considerable discussion before being held on December 1, 1969. Soon afterwards a pattern of unfairness in the results led to further publicity: those with birthdates later in the year seemed to have had more than their share of low lottery numbers and hence were more likely to be drafted. On January 4, 1970, the New York Times ran a long article, “Statisticians Charge Draft Lottery Was Not Random,” illustrated with a bar chart of the monthly averages. It described the way the lottery was carried out, and with hindsight one can see how the attempt at randomization broke down. The capsules were put in a box month by month, January through December, and subsequent mixing efforts were insufficient to overcome this sequencing.

The Wikipedia article [51] adds a little detail

The days of the year, represented by the numbers from 1 to 366 (including Leap Day), were written on slips of paper and the slips were placed in plastic capsules. The capsules were mixed in a shoebox and then dumped into a deep glass jar. Capsules were drawn from the jar one at a time.

The basic moral is that putting many items in a stationary container and then trying to mix them up is harder than you think.

Modern state lotteries put considerable effort into both being, and appearing to be, truly random. And they consult with professional statisticians to check that the results are xxx. Here is the entry on lottery machines from Wikipedia [56].

Most lotteries use mechanical lottery machines. These are more interesting to watch, and more transparent, both literally and figuratively: the audience can see exactly how the internal workings of the machine operate, and they can watch the balls come out of the machine; generally, the balls are visible during the entire draw.

A “gravity pick machine” has a drum with rotating arms inside; the drum itself may or may not rotate. A number of balls, each bearing a possible winning number, are dropped into the drum while it is spinning. A hole on the side or bottom of the drum allows the balls to drop onto a tray, one at a time (the tray is generally inclined so as to allow the numbers to go to the end and allow the other balls to fall into place). Machines of this type are used in Lotto type games, such as ‘Lotto 6-49’, Lotto 6/48 or other formulas and the American games Mega Millions, Hot Lotto, and Powerball.

A similar method is used to draw numbers from a manually-operated bingo cage; a weighted cup catches the ball, and drops it onto the tray as the cage is rotated. This variant was at one time used by both the Pennsylvania Lottery and the Illinois Lottery; in this variant, a cover is placed strategically, so as to obscure extra balls drawn beyond the number called for by the game matrix.

Air mix. Another type of lottery machine has a fan in the bottom the blows the balls around the inside of the container. The winning numbers go up and out through a tube. This is often used in “pick 3” and “pick 4” games, and also sees use in some “lotto” games; generally, either several tubes are in the container, or a single tube is used, and the ball is mechanically pushed onto a tray.

Though similar in style to the latter type of “air mix” machines, those currently used by the Powerball game do not use air mix; it is more in tune with a “gravity pick” machine, with the mixing paddles at the bottom of the machine. As the mixing paddle slows down, the ball lands on a pedestal, where it is carried to the top of the machine

and then pushed onto the tray. On rare occasions, a similar machine may be used in a "pick 3" or "pick 4" game.

xxx bingo uses blowers

xxx scrabble uses tiles in a small cloth bag

xxx Project: mixing for N business cards in a shaken box – how long do you need to shake?