Stat 155 Game theory, Yuval Peres **Fall 2004**

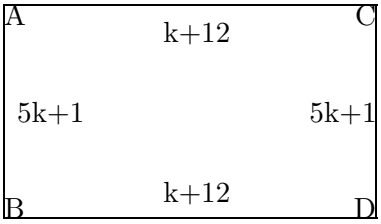
Homework 10

Question 1:

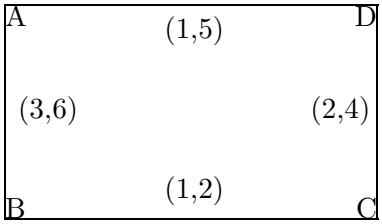
Find the Nash equilibria in the game of 'hawks and doves', whose payoffs are given by the matrix,

II / I	D	H
D	(1,1)	(0,3)
H	(3,0)	(-4,-4)

Question 2: a sequential congestion game. Six drivers will travel from A to D, each going via either B or C. The cost in travelling a given road depends on the number of drivers k that have gone before (including the current driver). These costs are displayed in the figure. Each driver moves from A to D in a way that minimizes his or her own cost. Find the total cost. Then consider the variant where a superhighway that leads from A to C is built, whose cost for any driver is 1. Find the total cost in this case also.



Question 3: simultaneous congestion. There are two drivers, one who will travel from A to C, the other, from B to D. Each road in the second figure has been marked (x,y), where x is the cost is any driver who travels the road alone, and y is the cost to each driver who travels the road along with the other. Note that the roads are travelled simultaneously, in the sense that a road is travelled by both drivers if they each use it at some time during their journey. Write the game in matrix form, and find all of the pure Nash equilibria.



Question 4: Sperner's lemma may be generalized to higher dimensions. In the case of $d = 3$, a simplex with four vertices (think of a pyramid) may be divided up into smaller ones. We insist that on each face of one of the small simplices, there are no edges or vertices of another. Label the four vertices of the big simplex $1, 2, 3, 4$. Label those vertices of the small simplices on the boundary of the big one in such a way that each such vertex receives a label of one of the vertices of the big simplex that lies on the same face of the big simplex. Prove that there is a small simplex whose vertices receive distinct labels.